Name: *Answers*

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Quiz No. 4

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 Find the derivative of $f(x) = e^{-\cos(x)}$.

$$f'(x) = \frac{d}{dx}\left(g\left(h(x)\right)\right) \quad , \text{ where}$$

$$g(x) = e^x \quad , \quad g'(x) = e^x$$

$$h(x) = -\cos(x) \quad , \quad h'(x) = \sin(x)$$

$$\Longrightarrow \quad f'(x) = g'\left(h(x)\right) \cdot h'(x)$$

$$= e^{-\cos(x)} \cdot \sin(x)$$

Problem 2 Find the derivative of $g(y) = \sec(4y^2 + 6))$.

$$g'(y) = \frac{d}{dx}(f(h(y))), \text{ where}$$

$$f(y) = \sec(y) \qquad , \qquad f'(y) = \sec(y)\tan(y)$$

$$h(y) = 4y^2 + 6 \qquad , \qquad h'(y) = 8y$$

$$\Rightarrow g'(y) = f'(h(y)) \cdot h'(y)$$

$$= \sec(4y^2 + 6)\tan(4y^2 + 6)\, 8y$$

Problem 3 Find the derivative of $h(z) = \csc^2(z)$.

$$h'(z) = \frac{\partial}{\partial z}\left(f(g(z))\right) \quad, \quad \text{where}$$

$$f(z) = z^2 \quad, \quad f'(z) = 2z$$

$$g(z) = \csc(z) \quad, \quad g'(z) = -\cot(z)\csc(z)$$

$$\Rightarrow h'(z) = f'(g(z)) \cdot g'(z)$$

$$= 2\csc(z)\left(-\cot(z)\csc(z)\right)$$

Problem 4   Using implicit differentiation, find $y'(x)$ given

$$xy^4 + yx^3 = (x+3)y$$

$\Rightarrow$ $x(y(x))^4 + y(x)x^3 = (x+3)y$

$\Rightarrow$ $\frac{d}{dx}\left(x(y(x))^4 + y(x)x^3 \right) = \frac{d}{dx}\left((x+3)y(x) \right)$

$(1)(y(x))^4 + x\left(4(y(x))^3 y'(x)\right) + y'(x)x^3 + y(x)(3x^2)$

$$= (1)y(x) + (x+3)y'(x)$$

$\Rightarrow$ $4x(y(x))^3 y'(x) + x^3 y'(x) - (x+3)y'(x) =$

$$- (y(x))^4 - 3x^2 y(x) + y(x)$$

$\Rightarrow$ $y'(x)\left(4x(y(x))^3 + x^3 - (x+3) \right) = - (y(x))^4 - 3x^2 y(x) + y(x)$

$\Rightarrow$ $y'(x) = \dfrac{-y^4 - 3x^2 y + y}{4xy^3 + x^3 - (x+3)}$